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LETTER TO THE EDITOR

Quantum correspondence of cyclotron and synchrotron radiation

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Abstract. The quantum origin of cyclotron and synchrotron radiation is considered. The radiation is assumed to be due to first-order transitions of electrons between Landau levels. A conflict is found in the following sense: (i) agreement between the classical and quantum emission coefficients (at large quantum numbers) is obtained by allowing the radial position of the guiding centre (label l) to change as a result of 'recoil' due to photon emission; (ii) the energy–angular momentum relationship in classical cyclotron radiation indicates that there could not have been any change in l. The difficulty is further highlighted by the fact that linear momentum conservation prevents the final photon from being emitted in an eigenstate of angular momentum.

In two earlier Letters (Lieu *et al* 1983, 1984) it was pointed out that, during cyclotron radiation, a non-trivial difficulty exists in the reconciliation of the basic conservation laws of energy, momentum, and angular momentum. In the present letter it will be shown that the same difficulty is also present in quantum mechanical considerations. The problem is seen in part during the search for correspondence between the quantum transition probability and the classical radiation rate. It appears more explicit when the conservation laws are formulated in terms of operators in the Schrödinger picture.

Controversy existed regarding the behaviour of the synchrotron radiation rate at high frequencies. A quantum mechanical calculation (Parzen 1951) reveals that the spectral emissivity differs from the classical result by a factor $\exp(-\lambda^2\omega^2/c^2)$ (where ω is the radiation angular frequency, and $\lambda^2 = \hbar c/eH$ where H is the magnetic field). At high photon energies this would lead to a drastic depletion of the emission rate as compared with the classical value—a phenomenon not observed in the laboratory. Later works (Judd *et al* 1952, Olsen and Wergeland 1952) revealed an 'error' in the calculations of Parzen (1951). Apart from the energy quantum number n (i.e. Landau level), the electron state is also characterised by a quantum number l (radial position of guiding centre, see Johnson and Lippmann 1949) which is a degeneracy. In Parzen (1951) the transition probability was computed for a $(n \to n', l \to l' = l)$ transition, whereas the other two papers showed that the cross-section must be summed over all the possible degeneracies l' which the electron can attain. Such a summation procedure removes the ultraviolet cutoff factor and the classical formula is once again obtained.

However, it must be pointed out that the price paid by the undertaking of these latter authors is a violation of basic conservation laws. The quantum mechanical emission of a photon is accompanied by transition of the electron $(n, l) \to (n', l')$ with the conservation of energy and angular momentum:

$$E_\gamma = (n-n')\omega_c = m\omega_c: \qquad J_z^\gamma = (n-l)-(n'-l') \tag{1}$$

since the angular momentum of the electron is $(n-l)$ (units $\hbar = 1$ are used). In Lieu *et al* (1983) it was shown that, in cyclotron radiation, the mth harmonic carries a total angular momentum $J_z^\gamma = m$. Use of this information in (1) then yields

$$l'-l=0. \tag{2}$$

Among the transition probability computations given earlier, only the procedure adopted by Parzen (1951) is consistent with (2).

The present work is not intended to scrutinise the correctness of such quantum calculations. The situation is, in fact, rather paradoxical. Quite apart from the fact that the allowance of a change in l leads to the correct radiation formula, it is difficult to imagine how l can stay fixed during the emission of a high-energy γ-photon. Clearly the guiding centre ought to respond to such a large transfer of linear momentum as explained in Lieu *et al* 1983).

This brings us to a consideration of linear momentum conservation in relation to the other conservation laws, from a quantum mechanical point of view. In Avron *et al* (1978) it was shown that translational invariance of the entire problem leads to an operator $\mathbf{\Pi}$ which commutes with the total Hamiltonian H of electron and radiation fields (including interaction), i.e.

$$[\mathbf{\Pi}, H] = 0.$$

In quantum mechanics we write the radiation (photon) momentum as $\boldsymbol{P} = \hbar \boldsymbol{k}$, so that the x and y components of Π^2 may be written in the following manner

$$\hat{\Pi}_x = m_e\omega_c(\hat{y}_0 + \lambda^2\hat{k}_x), \hat{\Pi}_y = -m_e\omega_c(\hat{x}_0 - \lambda^2\hat{k}_y)$$

where the '$\hat{\ }$' denotes operators in the Schrödinger Picture (and motion in the z direction is ignored). Following Lieu *et al* (1983), we then construct the operator $\hat{\Pi}^2$, given by

$$\hat{\Pi}^2 = (m_e\omega_c)^2[(\hat{x}_0^2 + \hat{y}_0^2) + 2\lambda^2(\hat{y}_0\hat{k}_x - \hat{x}_0\hat{k}_y) + \lambda^4(\hat{k}_x^2 + \hat{k}_y^2)] \tag{3}$$

and this operator is conserved in electromagnetic interactions.

Now the important point is that for the two-particle final state $\hat{\Pi}^2$ does *not* commute with the angular momentum operator of any individual particle. For the photon we can verify that†

$$[\hat{J}_z^\gamma, \hat{\Pi}^2] = [\hat{L}_z^\gamma, \hat{\Pi}^2] = -2\mathrm{i} m_e\omega_c\hbar^2(\hat{x}_0\hat{k}_x - \hat{y}_0\hat{k}_y)$$

where $\hat{J}_z$ and $\hat{L}_z$ are respectively total and orbital angular momentum of the photon (they differ by a spin operator, which commutes with $\hat{\Pi}^2$). Since the initial one-particle state (n, l) is an eigenstate of $\hat{\Pi}^2$, linear momentum conservation prevents the photon from being emitted in an eigenstate of $\hat{J}_z^\gamma$. The classical correspondence of such a conclusion is absurd, since we know that $\hat{J}_z^\gamma = m$ for the mth harmonic.

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† It is also easy to verify that $\hat{\Pi}^2$ does commute with the *total* angular momentum of the electron and photon, i.e. $[\hat{\Pi}^2, \hat{J}_z^e + \hat{J}_z^\gamma] = 0$.

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